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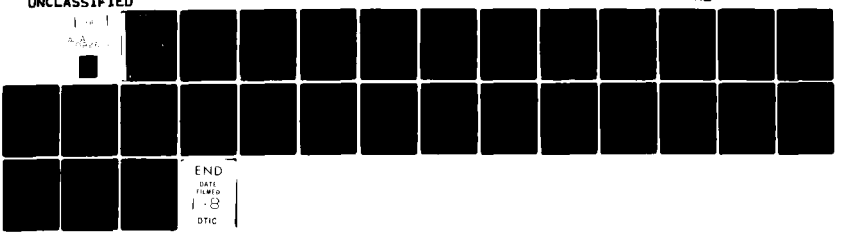

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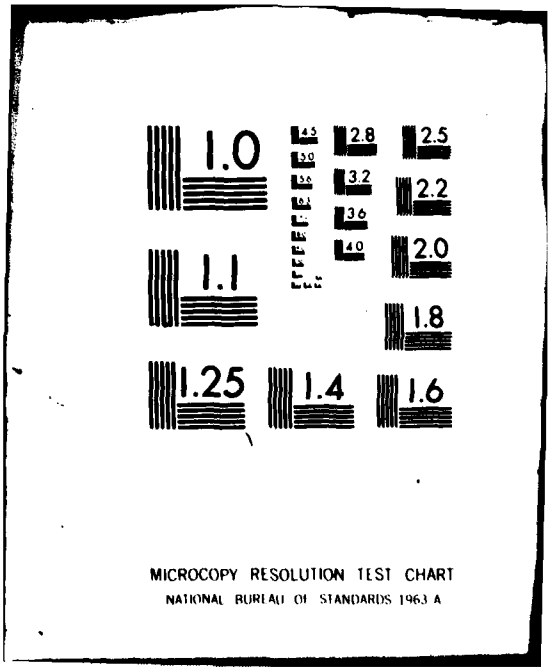
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EFFECTS OF COOPERATIVE ATOMIC BEHAVIOR ON LASERS

Final Technical Report

by

I.R. Senitzky

September 1980

EUROPEAN RESEARCH OFFICE

United States Army

London, England

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Cooperative behavior in a number of three-level atomic systems is analyzed. The atoms are coupled to two cavity modes resonant at the two intermediate frequencies and pumped coherently at the highest frequency. Three steady states are shown to exist; stability depends on pumping strength and cavity losses. Transition between steady states produces modulated field pulses in both modes; phase of modulation envelopes and fields is synchronized. Generation of various types of pulses is investigated. Generalizing to account for pump losses due to atomic reaction and their effect is studied.

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ABSTRACT

The effect of cooperative behavior, both with respect to pumping and relaxation, on a number of three-level atomic systems — which are assumed to have a dipole moment at all three transition frequencies — is analyzed. The atoms are coupled to two cavity modes resonant at the two intermediate frequencies and pumped coherently at the highest frequency. For sufficiently strong pumping, three steady states are shown to exist, the stability of which depends on the pumping strength and the cavity losses. Transition from one steady state to another produces modulated field pulses in both modes, with the phase of the modulation envelopes as well as the phase of the fields being synchronized. Conditions for the generation of various types of pulses are investigated. A generalization that takes into account pump losses due to atomic reaction is introduced and the effect of these losses is studied.

CONTENTS

I. Introduction

The operation of a three-level laser depends both on induced emission at one intermediate frequency and on incoherent relaxation at the other intermediate frequency. These two processes may be regarded as cooperative and noncooperative relaxation, respectively. It is the present purpose to analyze a novel and interesting type of possible behavior — qualitatively different from conventional laser-type behavior — in which the three-level atomic systems (hereafter referred to as molecules) undergo cooperative relaxation at both intermediate frequencies[1]. This may be accomplished by letting molecules for which all three transitions couple to the electromagnetic field interact with two (lossy) cavity modes tuned to the two respective intermediate frequencies. Labeling the three molecular energy levels in ascending order by $\hbar\omega_i$, $i = 1, 2, 3$, we consider a model in which N identical molecules are coupled to two modes with respective frequencies ω_{12}, ω_{23} and pumped at frequency ω_{13} , where $\omega_{ij} = |\omega_i - \omega_j|$. While such coupling, in accordance with well known selection rules, may be too weak in most atomic systems to produce appreciable effects, there exist atoms and molecules for which forbidden lines or overtones are sufficiently strong (such as the OCS molecule[2]), or for which two-photon pumping may be possible. The present discussion is not necessarily restricted to optical or infrared frequencies, but is also applicable to microwave frequencies.

In order to exhibit most simply the new features of the present system, we introduce the idealizations that the strength of coupling between molecule and mode depends only on the mode, and that transitions

other than those due to induced emission are negligible. The former is used widely in analyses of cooperative phenomena[3], and the latter will be discussed further. The formalism to be used is especially suitable for cooperative phenomena, and has the additional advantage of being interpretable both quantum mechanically and classically[4,5]. The Hamiltonian for the complete system is described by the following expressions:

$$
\begin{aligned}
H &= H_o + H_{12} + H_{23} + H_{13} , \\
H_o &= \sum_{i=1}^{3} h\omega_i a_i^\dagger a_i , \\
H_{12} &= h\gamma_{12}(A_1 A_2^\dagger B_{12} + B_{12}^\dagger A_2 A_1^\dagger) , \\
H_{23} &= h\gamma_{23}(A_2 A_3^\dagger B_{23} + B_{23}^\dagger A_3 A_2^\dagger) , \\
H_{13} &= -ih(WA_1 A_3^\dagger - W^* A_3 A_1^\dagger) , \\
a_j(t) &= A_j(t) e^{-i\omega_j t} , \\
b_{jk}(t) &= B_{jk}(t) e^{-i\omega_{jk} t} , \\
B_{jk}(t) &= B_o^{(jk)}(t) - i\gamma_{jk} \int_o^t dt_1 A_j^\dagger(t_1) A_k(t_1) e^{-\xi_{jk}(t-t_1)} , \quad j < k .
\end{aligned}
\tag{1.1}
$$

In these relationships, the $a_i^\dagger$'s and a_i's are, quantum mechanically, (boson) creation and annihilation operators for molecules in the i'th level, and classically (complex) harmonic oscillator amplitudes. The $b_{jk}^\dagger$'s and b_{jk}'s are photon creation and annihilation operators for the field in the jk mode (the mode that resonates at frequency ω_{jk}), the order

of the indices having no significance. $B_o^{(jk)}$ describes the field of the jk mode in the absence of the molecules. W refers to the pump field. It is a prescribed constant, and has the dimensions of frequency, with $2|W|$ being the Rabi frequency. Description of the effect of the pump field in a manner similar to that of the mode fields, by a complex amplitude, would involve the replacement of W by $i\gamma_{13}B_{13}$. The integral expression for B_{jk} in terms of $A_j^\dagger$ and A_k is a result of the solution of the equations of motion for the field in terms of the molecular variables, with ξ_{jk} being the loss constant of the jk mode[4]. (Energy of the mode uncoupled from the molecules decays as $\exp(-2\xi_{jk}t)$]. The commutation relationships (or Poisson brackets multiplied by i, for classical variables) are

$$[A_i, A_i^\dagger] = 1 , \qquad [B_{ij}, B_{ij}^\dagger] = 1 , \tag{1.2}$$

all other (equal-time) commutators vanishing.

II. Equations of Motion

The molecular equations of motion that we obtain from the above Hamiltonian are given by

$$\begin{aligned} \dot{A}_1 &= W^* A_3 - i\gamma_{12} B_{12}^\dagger A_2 , \\ \dot{A}_2 &= - i\gamma_{23} B_{23}^\dagger A_3 - i\gamma_{12} A_1 B_{12} , \\ \dot{A}_3 &= - W A_1 - i\gamma_{23} A_2 B_{23} . \end{aligned} \tag{2.1}$$

One sees immediately that these equations are consistent with the normalization

$$\Sigma A_i^\dagger A_i = N , \tag{2.2}$$

where, as stated previously, N is the number of molecules under consideration. The normalization condition is equivalent to a statement concerning the cooperation of the molecules, and implies that we are considering a situation of maximum molecular cooperation[4,5]. In the case of less than maximum cooperation, N may be regarded as a cooperation number.

We consider only the case of sufficiently damped cavity modes, so that the mode-fields follow the respective resonant polarizations adiabatically. The last of Eqs. (1.1) yields for this case

$$B_{jk} = B_o^{(jk)} - i(\gamma_{jk}/\xi_{jk})A_j^\dagger A_k . \tag{2.3}$$

In view of the fact that our interest lies in macroscopic — rather than microscopic — phenomena, the classical description is a good approximation under most conditions[5], and will be used henceforth. All the variables in the above equations may now be regarded as c-numbers, and $B_o^{(jk)}$ is to be taken to be zero.

Eliminating the field from the molecular equations of motion, and setting

$$x_i = A_i/N^{1/2}, \quad \tau = Wt , \quad c_{ij} = N\gamma_{ij}^2/\xi_{ij}W , \tag{2.4}$$

one obtains

$$
\begin{aligned}
x_1' &= x_3 + c_{12} x_1 |x_2|^2 , \\
x_2' &= c_{23} x_2 |x_3|^2 - c_{12} |x_1|^2 x_2 , \\
x_3' &= - x_1 - c_{23} |x_2|^2 x_3 ,
\end{aligned}
\tag{2.5}
$$

the prime indicating differentiation with respect to τ. Clearly, the phase of x_2 is constant, and is given by its initial value. Noting that the normalization condition (2.2) now reads,

$$
\Sigma |x_i|^2 = 1 , \tag{2.6}
$$

we eliminate $|x_2|^2$ from the atomic equations of motion for x_1 and x_3 to obtain

$$
\begin{aligned}
x_1' &= x_3 + c_{12} x_1 (1 - |x_1|^2 - |x_3|^2) \\
x_3' &= - x_1 - c_{23} x_3 (1 - |x_1|^2 - |x_3|^2) .
\end{aligned}
\tag{2.7}
$$

We have here two equations for two complex variables. If these are known, $|x_2|$ can be obtained from Eq. (2.6), with the phase of x_2 being its initial phase. Instead of working with two complex variables, it is convenient to introduce the four real variables

$$
\begin{aligned}
n_1 &= |x_1|^2 , \qquad n_3 = |x_3|^2 \\
u &= x_1^* x_3 + x_3^* x_1 , \\
v &= -i(x_1^* x_3 - x_3^* x_1) .
\end{aligned}
\tag{2.8}
$$

It is seen immediately that

$$u^2 + v^2 - 4n_1n_3 = 0 , \tag{2.9}$$

so that only three of these variables are independent. The reason is, of course, the fact that only the relative phase of x_1 and x_3 affects these variables. The equations of motion for n_1, n_2, u, v, are obtained from Eqs. (2.7) in the form

$$\begin{aligned} n_1' &= u + 2c_{12}n_1(1-n_1-n_3) , \\ n_3' &= - u - 2c_{23}n_3(1-n_1-n_3) , \\ u' &= 2(n_3-n_1) + (c_{12} - c_{23})u(1-n_1-n_3) , \\ v' &= (c_{12} - c_{23})v(1 - n_1 - n_3) . \end{aligned} \tag{2.10}$$

These equations describe all the physically meaningful quantities of the present problem, since only the relative phase — rather than the absolute phase — of the amplitude A_1 and A_2 (as well as their absolute values, of course) has physical significance. Thus, there should be only three independent (real) quantities that describe the atomic system. It is useful to note that

$$u^2 \leq 4n_1n_3 , \quad v^2 \leq 4n_1n_3 , \quad n_1 + n_2 \leq 1 . \tag{2.11}$$

The energy in the cavity modes due to the atomic polarization is given by

$$\hbar\omega_{12}|B_{12}|^2 = \hbar\omega_{12}\left(\frac{\gamma_{12}}{\xi_{12}}\right)^2 N^2n_2(1-n_1-n_3) \tag{2.12}$$

for the (1.2) cavity, and

$$h\omega_{23}|B_{23}|^2 = h\omega_{23}\left(\frac{\gamma_{23}}{\xi_{23}}\right)^2 N^2 n_2(1-n_1-n_3) \tag{2.13}$$

for the (2.3) cavity.

III. Solution of the Equations of Motion

There exist several steady-state solutions of Eqs. (2.10), which we label for convenience.

Solution Z :

$$n_1 = n_2 = u = v = 0 \ . \tag{3.1}$$

In this steady-state, the entire atomic population is in the second energy level. The conditions for stability (in the sense of assymptotic orbital stability[6]) of this solution are

$$(c_{12}/c_{23}) < 1 \ , \qquad c_{12}c_{23} < 1 \ . \tag{3.2}$$

Solution S :

This solution exists only for

$$c_{12}c_{23} > 1 \ , \tag{3.3}$$

and is given by

$$n_1 = [1 - (c_{12}c_{23})^{-1/2}] \frac{c_{23}}{c_{12} + c_{23}} ,$$

$$n_3 = [1 - (c_{12}c_{23})^{-1/2}] \frac{c_{12}}{c_{12} + c_{23}} \qquad (3.4)$$

$$u = -2 \frac{(c_{12}c_{23})^{1/2} - 1}{c_{12} + c_{23}} , \quad v = 0 .$$

The stability condition for this solution is

$$(c_{12}/c_{23}) < 1 . \qquad (3.5)$$

Solution C :

$$n_1 = n_3 = \frac{1}{2} , \quad u = 0 , \quad v = \pm 1 . \qquad (3.6)$$

In this steady state, the entire population is evenly distributed between the first and third level, with none in the second. This state is similar to that of a two-level-system. In fact, Solution C is a special case of a class of solutions for which $n_1 + n_3 = 1$, and which may be regarded as quasi-steady-state solutions, since $n_2 = 0$. We label these as follows:

Solution P :

$$n_1(\tau) = \frac{1}{2} + [n_1(0) - \tfrac{1}{2}]\cos 2\tau + \tfrac{1}{2} u(0) \sin 2\tau ,$$

$$n_3(\tau) = \frac{1}{2} - [n_1(0) - \tfrac{1}{2}]\cos 2\tau - \tfrac{1}{2} u(0) \sin 2\tau , \qquad (3.7)$$

$$u(\tau) = u(0) \cos 2\tau - 2[n_1(0) - \tfrac{1}{2}] \sin 2\tau ,$$

$$v(\tau) = v(0) = \{4n_1(0)[1 - n_1(0)] - u^2(0)\}^{1/2} .$$

It is seen that solution C corresponds to a special choice of initial conditions in these equations.

The stability of Solution P can be divided into two kinds: stability with respect to a perturbation that changes one solution into another of the same class (with $n_2 = 0$), and stability with respect to a perturbation that changes a solution of this class into one outside the class (into one for which $n_2 > 0$). The first kind of stability does not depend on c_{12} and c_{23}, as can be seen from Eqs. (2.10), which show that the c-terms vanish for $n_1 + n_3 = 1$, and also, that $n_1 + n_3$ remains constant if it is equal to unity initially. Furthermore, Eqs. (3.7) show that any perturbation that leaves the system in a condition that can be an initial condition for a P solution will oscillate periodically. Thus a small perturbation of the first kind produces only a small change in the solution, but there exists no return to the original solution. On the other hand, a perturbation of the second kind, that results in a reduction of $n_1 + n_3$, will be followed by either a return to a P solution or to a qualitative change in the state. The condition that a P solution returns to the class of P solutions for an arbitrary perturbation is expressed by

$$c_{12}/c_{23} > 1 . \tag{3.8}$$

The regions of the stability of the several types of steady-state (or quasi-steady-state) solutions in the c_{12}, c_{23} plane are shown in Fig. 1. The similarity of the figure to a phase diagram is obvious. If n_2 is considered (formally) to be an "order parameter", then the transition $S \leftrightarrow Z$ corresponds to a second-order phase transition, while $S \leftrightarrow P$ and $Z \leftrightarrow P$ correspond to first-order phase transitions.

It should be noted that if v vanishes at any time, it will remain zero subsequently. From the inequalities (2.11) it follows that if n_1 or n_3 vanishes at any one time, so will v at that particular time, and, therefore, also subsequently. The solutions for which $v = 0$ form, thus, a group of solutions which can be studied separately. We proceed to do so.

One notes, first, that the vanishing of v implies that x_1 and x_3 may be taken to be real, since the initial phase of one atomic amplitude, say x_1, may be chosen arbitrarily, and the choice of $x_1(0)$ to be real together with the vanishing of $v(0)$ requires $x_3(0)$ to be real. Equations (2.7) become, therefore, two equations for two real quantities. Setting, for simplicity of notation,

$$x_1 = x , \quad x_3 = y , \quad c_{12} = c_1 , \quad c_{23} = c_2 , \tag{3.9}$$

we can rewrite Eqs. (2.7) as

$$\begin{aligned} x' &= y + c_1 x(1 - x^2 - y^2) \\ y' &= - x - c_2 y(1 - x^2 - y^2). \end{aligned} \tag{3.10}$$

It is useful to introduce polar coordinates, with

$$x = r \cos \theta , \qquad y = r \sin \theta , \tag{3.11}$$

which transform the equations into

$$\begin{aligned} r' &= r(1 - r^2)(c_1 \cos^2\theta - c_2 \sin^2\theta) , \\ \theta' &= - 1 - \tfrac{1}{2}(c_1 - c_2)(1 - r^2)\sin 2\theta . \end{aligned} \tag{3.12}$$

The steady-state solutions S and Z remain solutions of the present set of equations, since $v = 0$ for these solutions. In the present notation we have, for solution S ,

$$r^2 = 1 - (c_1 c_2)^{-1/2} , \qquad \tan\theta = - (c_1/c_2)^{1/2} , \tag{3.13}$$

and, for solution Z ,

$$r = 0 . \tag{3.14}$$

Solution P is now a single solution given by

$$r = 1 , \qquad \theta = - \tau + \theta_o , \tag{3.15}$$

where θ_o is an arbitrary constant determined by the initial conditions. In the terminology of nonlinear equation[6], Z and S are singular points, and P is a limiting cycle. The only steady state in which the cavity modes are excited is S . As mentioned previously, the total population is in the middle level in Z , while in P , all the molecules undergo a Rabi oscillation between the first and third levels.

The most interesting behavior is exhibited by the time dependent solutions. A convenient method of discussing the solutions is the consideration of the motion of the point representing a solution along trajectories in the (x,y) — or (r,θ) — plane. Before discussing the solutions for arbitrary c_1 and c_2 , it is useful to investigate the case $c_1 = c_2 = c$. For this case, an analytic expression for the trajectories along which the point representing the solution travels is given by

$$r^2 = 1 - K \exp(cr^2 \sin 2\theta) , \tag{3.16}$$

where K is a constant of integration determined by the initial conditions. This equation describes a family of closed curves as shown in Fig. 2: in (a), $c = 1.5$, and in (b), $c = 0.5$. Each curve corresponds to a different K. $K = 0$ corresponds to solution P, $K = 1$ corresponds to solution Z as well as to the separatrix[6] for $c > 1$ (the dotted curve), and $K = c^{-1}\exp(c-1)$ (for $c > 1$) corresponds to solution S. The point representing the solution moves along the trajectory in a clockwise sense. We see that all solutions are periodic, except for those corresponding to the singular points and to the separatrix. The period is of the order of a Rabi cycle π/W. It should be noted that only bilinear terms in x and y have physical significance, so that reflection in the origin leaves all physical quantities unchanged.

For $c_1 \neq c_2$, one can see from the polar-coordinate form of the equations of motion that the trajectory of a given solution will spiral in or spiral out (if the initial state is not a steady state) depending on whether $c_2 > c_1$ or $c_1 > c_2$, approaching the stable steady state while following roughly the outline of the periodic trajectories of Fig. 2. Two computer graphs of such solutions are shown in Fig. 3 for the following conditions: (a) $c_1 = 1.4$, $c_2 = 1.6$, $x(0) = 0.95$, $y(0) = 0$; (b) $c_1 = 0.7$, $c_2 = 0.4$, $x(0) = 0.05$, $y(0) = 0$.

IV. Cavity Fields

The field energy in the modes is given by Eqs. (2.12) and (2.13). In the present notation, we have

$$\hbar\omega_{12}|B_{12}|^2 = \hbar\omega_{12}\left(\frac{\gamma_{12}}{\xi_{12}}\right)^2 N^2x^2(1-r^2) , \qquad (4.1)$$

for the energy in the (1,2) mode, and

$$\hbar\omega_{23}|B_{23}|^2 = \hbar\omega_{23}\left(\frac{\gamma_{23}}{\xi_{23}}\right)^2 N^2 y^2 (1-r^2) , \qquad (4.2)$$

for the energy in the (2,3) mode. One sees that the fields are modulated as the point representing the solution moves along a spiral, the modulation frequency being approximately $2W$, with a definite phase relationship being maintained between the modulation envelopes for the two modes.

A sudden change in W, that is, in the pump strength, for $c_1 < c_2$, which produces a crossing of the curve $c_1 c_2 = 1$ (see Fig. 1) will produce a transition from S to Z (a modulated decay of the field of the two modes) for an increase in pump strength, and produce a transition from Z to S (a modulated rise of the fields of the two modes) for a decrease in pump strength. The $S \to Z$ transition is caused by a disappearance of S, while the $Z \to S$ transition is caused by an (assumed) arbitrarily small perturbation of the (unstable) Z state.

A particularly interesting effect is the occurrence of a modulated pulse in both modes as a result of a $Z \to P$ transition, when stability is shifted from Z to P, or vice versa, by a crossing of the line $c_1 = c_2$ (a line of "bifurcation points"[6]). Such a transition can be produced by a change in ξ_{12} or ξ_{23}, the cavity loss, conceivably by electronic methods. The $Z \to P$ transition is illustrated in Fig. 3b (for a perturbed Z); the $P \to Z$ transition is a spiralling-in from $r = 1 - \epsilon$ to $r = 0$. A period of revolution along the spiral is approximately $2\pi/W$, and the pitch of the spiral near a given r varies as $c_1 - c_2$. A $P \to S$ transition is illustrated in Fig. 3a.

The only steady-state for which the two modes are excited is the S state. For the other steady states, the cavity fields vanish. However, in a transition from one zero-field state to another zero-field state, such as the transition $Z \rightarrow P$ illustrated in Fig. 3b, a transient pulse is excited in both cavities. The pulse in both modes is modulated, with the phases of the fields and the modulation envelopes for the two modes being correlated. The transition $P \rightarrow S$ will produce a modulated rise in the fields from zero to the steady-state value corresponding to the state S, and the transition $S \rightarrow Z$ or $S \rightarrow P$ will produce a modulated decay in the field. It should be noted that a change in pump strength is represented by a change in the radius vector in the c_1, c_2 plane, the angle being kept constant.

V. Effect of Losses in Pump Mode

In the preceding analysis, the pump field was considered prescribed. It is interesting to investigate a situation in which the effect of the atomic system on the pump field is taken explicitly into account. Such an effect may be considered to be significant when the pump field is sufficiently weak. In order to incorporate this effect into the analysis, we consider a model in which the pump drives a cavity mode that is coupled to the (1,3) transition in the same manner as the (1,2) and (2,3) modes are coupled to the transitions at their respective frequencies. Thus, the only difference between the formal treatment of the (1,3) mode and the other two modes should be the inclusion of a prescribed part for the field

of the (1,3) mode, that is, the inclusion of a nonvanishing B_o in expressions such as Eq. (2.3), or the last of Eqs. (1.1). We therefore replace H_{13} in the Hamiltonian of Eqs. (1.1) by

$$\tilde{H}_{13} = h\gamma_{13}(A_1 A_3^\dagger B_{13} + B_{13}^\dagger A_3 A_1^\dagger) \tag{5.1}$$

with

$$B_{13} = B_o - i\frac{\gamma_{13}}{\xi_{13}} A_1^\dagger A_3 \ . \tag{5.2}$$

As mentioned earlier, the relationship between the prescribed effect of the pump, described by W($|W|$ being one half the Rabi frequency), and a complex field amplitude B_{13} is given by

$$W = i\gamma_{13} B_{13} \ . \tag{5.3}$$

B_o describes the pump field in the absence of the molecules.

The molecular equations of motion can now be written as

$$\begin{aligned} \dot{A}_1 &= -i\gamma_{13} B_{13}^\dagger A_3 - i\gamma_{12} B_{12}^\dagger A_2 \ , \\ \dot{A}_2 &= -i\gamma_{12} A_1 B_{12} - i\gamma_{23} B_{23}^\dagger A_3 \ , \\ \dot{A}_3 &= -i\gamma_{13} A_1 B_{13} - i\gamma_{23} A_2 B_{23} \ , \end{aligned} \tag{5.4}$$

which, after substitution for the fields in terms of the driving field B_o and the molecular radiation, become, in the present classical formalism

$$x_1' = x_3 + x_1[\tilde{c}_{12}(1 - |x_1|^2 - |x_3|^2) + \tilde{c}_{13}|x_3|^2] ,$$

$$x_3' = - x_1 - x_3[\tilde{c}_{23}(1 - |x_1|^2 - |x_3|^2) + \tilde{c}_{13}|x_1|^2] , \tag{5.5}$$

where $\tilde{c}_{ij}$ is defined as

$$\tilde{c}_{ij} = \frac{N\gamma_{ij}}{\xi_{ij}W_o} \tag{5.6}$$

with

$$W_o = \gamma_{13}|B_o| , \tag{5.7}$$

while τ is now defined as $W_o t$. We see that the new equations of motion differ from the previous ones [Eqs. (2.7)] in that W is replaced by W_o and the $\tilde{c}_{13}$ terms have been added. For convenience of notation the tildes will be suppressed where there is no possibility of confusion. In terms of the real variables n_1, n_3, u, v , the equations become

$$\begin{aligned} n_1' &= u + 2n_1[c_{12}(1 - n_1 - n_3) + c_{13}n_3] , \\ n_3' &= - u - 2n_3[c_{23}(1 - n_1 - n_3) + c_{13}n_1] , \\ u' &= 2(n_3 - n_1) + [(c_{12} - c_{23})(1 - n_1 - n_3) + c_{13}(n_3 - n_1)]u , \\ v' &= [(c_{12} - c_{23})(1 - n_1 - n_3) + c_{13}(n_3 - n_1)]v . \end{aligned} \tag{5.8}$$

These equations are more complicated, of course, than the previous equation of motion. As an illustration of this fact, it can be noted that for the much simpler case $c_{12} = c_{23} = 0$, that is, in the absence

of a middle level, the solution of the equations is by no means trivial, and has been discussed in the literature[7]. Presently, we investigate only the steady-state solutions.

Our interest lies in the modification, if any, of the present effects produced by the reaction of the atoms on the pump. We assume that this reaction is small compared to the interactions that are responsible for the effects under present consideration, this smallness being described quantitatively by

$$c_3 < c_1 + c_2 \ . \tag{5.9}$$

The absence of reaction to the pump corresponds to $c_3 = 0$. The steady state solutions, as derived formally from the equation of motion [Eqs. (4.8)] are

Solution Z:

$$n_1 = n_3 = u = v = 0 \ . \tag{5.10}$$

Solution S, for $c_1 c_2 \geq 1$ and $(c_1 + c_2)(c_1 c_2)^{-1/2} > c_3$ only:

$$n_1 = [1 - (c_1 c_2)^{-1/2}] \frac{c_2}{c_1 + c_2 - c_3} \ ,$$

$$n_3 = [1 - (c_1 c_2)^{-1/2}] \frac{c_1}{c_1 + c_2 - c_3} \ , \tag{5.11}$$

$$u = -2 \frac{(c_1 c_2)^{1/2} - 1}{c_1 + c_2 - c_3} \ , \qquad v = 0 \ .$$

Solution C , for $c_3 \leq 2$ only:

$$n_1 = n_3 = \frac{1}{2} , \; u = -\frac{1}{2} c_3 , \qquad v = \pm (1 - \frac{1}{4} c_3^2)^{1/2} . \tag{5.12}$$

Solution R , for $c_3 \geq 2$ only:

$$n_1 = \frac{1}{2} \pm \frac{1}{2} (1 - \frac{4}{c_3^2})^{1/2} , \quad n_3 = \frac{1}{2} \mp \frac{1}{2} (1 - \frac{4}{c_3^2})^{1/2},$$
$$u = -\frac{2}{c_3} , \qquad v = 0 . \tag{5.13}$$

There exists, also (as previously), a class of solutions for which $n_1 + n_3 = 1$. We can refer to these as two-level solutions. Solutions C and R are special, steady state, members of this class. The equations of motion show that these solutions are unaffected by c_1 and c_2 , and are determined only by c_3 . As mentioned above, they have been discussed elsewhere[7]. However, the stability of these solutions with respect to a perturbation that reduces $n_1 + n_3$ does depend on c_1 and c_2 , in general, since the full equations of motion, including the c_1 and c_2 terms, determine this stability. We will refer to the class of two-level solutions as P solutions, just as we did for the case of vanishing c_3 .

An analysis of the stability of the present solutions is more complicated, of course, than the analysis for the case $c_3 = 0$. One obtains the following results.

Stability of Solution Z:

Z is stable for $c_1 < c_2$ and $c_1 c_2 < 1$; it is unstable otherwise. As far as Z is concerned, the stability conditions are unaffected by a nonvanishing c_3 .

Stability of Solution S:

S is stable for $c_1 < c_2$ and unstable for $c_1 > c_2$. While here, too, the stability condition does not involve c_3, the conditions for the existence of the solution do involve c_3.

Stability of Solution C:

As in the case of vanishing c_3, one can consider two kinds of stability here. C belongs to the class of P solution for which $n_2 = 0$, that is, for which only the first and third levels are populated. It can be shown[7] that, for $c_3 < 2$, the class of P solutions does not exhibit either stability (in the sense of assymptotic orbital stability[6]) or instability; a perturbation that takes one solution of this class into another remains periodic in time. However, a perturbation that produces a change in n_2 (or in $n_1 + n_3$) will either decay or increase (initially) with time. With respect to such a perturbation, solution C is stable for $c_1 > c_2$ and unstable for $c_1 < c_2$. The same conditions apply to all members of the class of P solutions for $c_3 < 2$.

Stability of solution R:

As stated above, R exists only for those values of c_3 such that $c_3 > 2$. For $c_1 > c_2$, R is stable. For $c_1 < c_2$, R is stable only for $c_3 > (c_1 + c_2)(c_1c_2)^{-1/2}$. In other words, in the range of c_3 such that

$$2 < c_3 < (c_1 + c_2)(c_2c_2)^{-1/2} , \tag{5.14}$$

R is stable only if $c_1 > c_2$. It is interesting to note that this stability condition complements the stability condition for S. For those values of c_1 and c_2 for which both S and R exist, S is stable where R is unstable, and vice versa. It should be noted, further, that for sufficiently weak pumping, R is the only stable state in all regions of the c_1, c_2 plane; that is, as the pump becomes sufficiently weak, only the first and third levels participate in the interaction with the pump, regardless of the relationship between c_1 and c_2.

Atomic relaxation and spontaneous emission have been ignored. These will produce the perturbations of the unstable steady state that result in some of the transient pulses described above, and perturbation of the stable steady states. One may reasonably expect that after the beginning of a transient mode-excitation, the induced emission will become dominant and other relaxation effects less important.

VI. Summary and Conclusions

Coherently pumped three-level atomic systems coupled to two resonant cavity modes at the intermediate frequencies have been analyzed by means of an idealized model. It was shown that the system has novel and unusual features, such as the ability to generate synchronized pulses at both frequencies as well as modulated steady-state oscillation. The nature of the output can be controlled both by the strength of the pump and by the losses in the cavities.

The significant idealization used in the present discussion is the neglect of atomic relaxation other than by coupling to the modes, and the neglect of spatial effects in the coupling between atoms and field. These idealizations should be eliminated in further theoretical analysis before an experimental investigation of the interesting – and, potentially useful – effects described above is undertaken.

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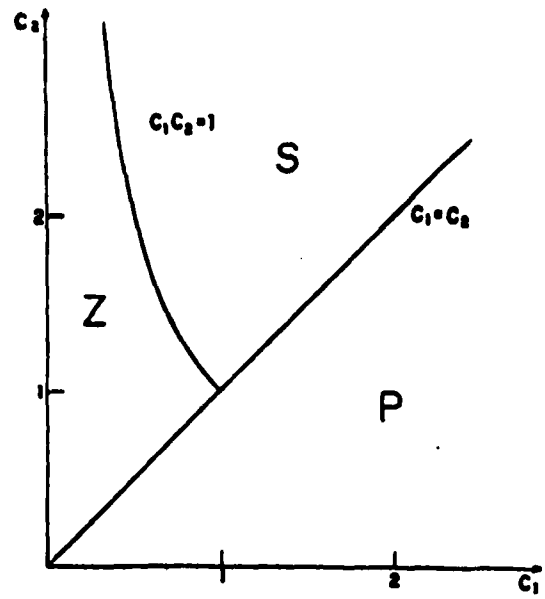

Fig. 1

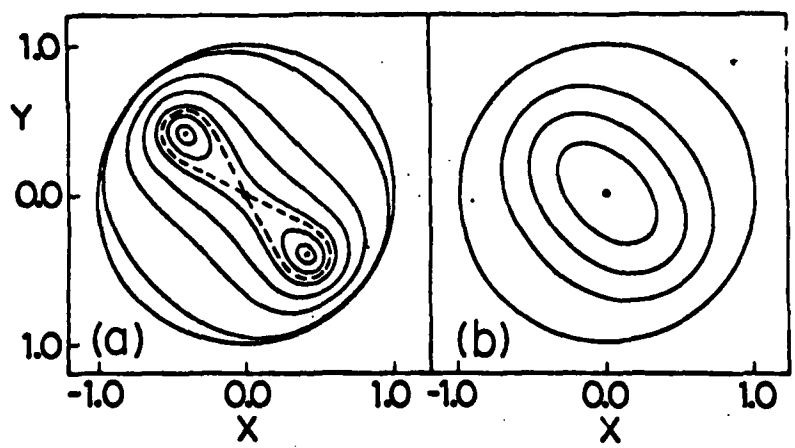

Fig. 2

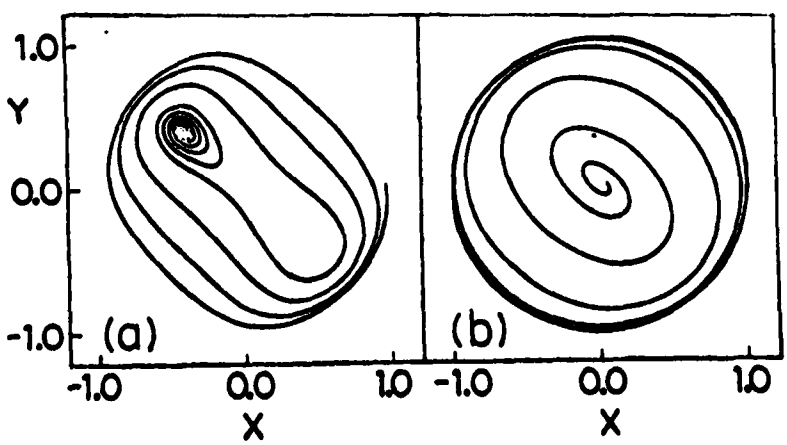

Fig. 3

Publications resulting from work on Grant No. DAERO-77-G-077, as of October 1980:

I.R. Senitzky and Jan Genossar, "Cooperative Relaxation in Coherently Pumped Three-Level Systems", Physical Review Letters 44, 1453 (1980).

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